**Year 12 Examination, 2018**

**Question/Answer Booklet**

**MATHEMATICS SPECIALIST**

**Section Two: Calculator-assumed**

Student Name/Number: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Teacher Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Time allowed for this section**

Reading time before commencing work: ten minutes

Working time for this section: one hundred minutes

**Materials required/recommended for this section**

**To be provided by the supervisor:** This Question/Answer Booklet

Formula Sheet (retained from Section One)

**To be provided by the candidate:**

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

**Important note to candidates**

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

**Structure of this paper**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of exam |
| Section One: Calculator-free | 7 | 7 | 50 | 54 | 35 |
| Section Two: Calculator-assumed | 13 | 13 | 100 | 100 | 65 |
|  | | | | | 100 |

**Instructions to candidates**

1. The rules for the conduct of School exams are detailed in the *\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_School/College assessment policy*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.

1. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   * Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   * Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.
2. **Show all working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
3. It is recommended that you **do not use pencil**, except in diagrams.
4. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

**Section Two: Calculator-assumed 65% (100 Marks)**

This section has **13** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

* Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
* Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Suggested working time: 100 minutes.

**Question 8 (5 marks)**

Solve algebraically .

.

**Question 9 (6 marks)**

1. Given that

 ,

express  in Cartesian form. (3 marks)

1. Use your calculator to express  in Cartesian form, giving your answer correct to eight decimal places. (1 mark)
2. Determine the smallest positive value  such that . (1 mark)

.

(d) Hence, or otherwise, determine the exact value found in part (b). (1 mark)

**Question 10 (6 marks)**

It is given that that  is a solution of the quartic equation



in which  and  are real numbers.

(a) Determine the values of and . (2 marks)

(b) Factorise the quartic as a product of two quadratics and hence deduce the other three solutions of the quartic equation. (4 marks)

**Question 11 (9 marks)**

The planes and have Cartesian equations

and .

The planes and intersect in the line L.

(a) Evaluate the cross product . (1 mark)

(b) Use a geometrical argument to explain why is parallel to L. (2 marks)

(c) Determine a vector equation for L given that the point (0,1,0) lies on this line. (2 marks)

(d) The plane has the Cartesian equation

for some constants and .

1. What are the values of and if the planes , and have infinitely points in common? (3 marks)
2. What are the values of and if the planes , and have no point in common? (1 mark)

**Question 12 (7 marks)**

Given  and  , determine:

(a)  and  (2 marks)

(b) the domain and range of  (2 marks)

(c) the domain and range of  (3 marks)

**Question 13 (5 marks)**

The curve and the line  meet at .

1. Determine the area bounded by the curve, the line and the axis. (3 marks)
2. Calculate the volume formed when this area is rotated about the x-axis. (2 marks)

**Question 14 (11 marks)**

A particle moves along a straight line. The displacement of the particle from the origin is  cm and its velocity is  cms-1. The particle is moving such that.

(a) Determine the acceleration of the particle and hence show that the motion of the particle is simple harmonic with period . (2 marks)

(b) Given the initial position of the particle was at the centre of oscillation, determine an expression for the displacement of the particle as a function of time. (2 marks)

(c) Find the maximum and minimum speed of the particle. (2 marks)

(d) Determine the distance travelled by the particle during the fourth second correct to two decimal places.

(3 marks)

(e) Is the particle travelling towards or away from the initial position at  seconds? Justify your answer.

(2 marks)

**Question 15 (4 marks)**

Use partial fractions to prove that



**Question 16 (4 marks)**

Consider the parabola P defined by  over the range 

1. Show that the line L given by meets the parabola at  and another point  if . (1 mark)
2. Determine the value of  if the area contained between L and P over  equals the area contained between them for . (3 marks)

**Question 17 (15 marks)**

People living a certain city are susceptible to migraine attacks. The number of migraines suffered by each person in any given week is known to be normally distributed, with a mean 5.04 and a standard deviation of 1.8.

1. What proportion of people suffer between 5 and 6 migraines per week? (2 marks)
2. Calculate the upper quartile of the distribution. (2 marks)
3. Researchers plan to test the effectiveness of acupuncture as treatment for migraine. They began by using acupuncture on a small random sample, and they found that for people in this sample the average number of migraines per week was 3.81, with a standard deviation of 1.3.

They now wish to use a larger sample to estimate the average number of migraines per week that would be suffered by people treated with acupuncture, to an accuracy of 0.2 at a 95% level of confidence.

How large does the sample need to be? Use the standard deviation from the preliminary trial to make your estimate. (2 marks)

1. In a random sample of 200 people treated with acupuncture it is found that the average number of migraines is 4.55, with a standard deviation of 1.36.

Calculate a 95% confidence interval for the average number of migraine attacks suffered by people treated with acupuncture. (2 marks)

1. Comment on the claim that ‘the testing clearly shows that acupuncture reduces the frequency of migraine’ attacks. (2 marks)
2. Researchers also tested an alternative treatment for migraine attacks. They found that for the 100 hundred randomly selected people who were given a certain drug, the average number of migraine attacks was 4.18, with a standard deviation of 2.43.

Use a confidence interval to test the claim that ‘these tests clearly show that the drug is more effective than acupuncture for treating migraine attacks’. (5 marks)

**Question 18 (9 marks)**

Two small rockets are launched simultaneously; rocket A from the origin of coordinates O, and the rocket B from a point P located 600 m the east of O.

The initial velocity of rocket A is given by and throughout its flight its acceleration is **.**

The velocity of rocket B is the constant vector where the constant is the speed and the constant is the angle of elevation.

You may assume that the unit vectors and are aligned in the traditional manner, i.e. points East and points North in a horizontal plane H, and points vertically upwards. You may also assume that distances are measured in metres and time is measured in seconds.

(a) How far from the origin O is rocket A when it returns to the plane H? (4 marks)

(b) Determine the value of if the flight paths of the two rockets meet. (3 marks)

(c) Determine the speed of rocket B if the rockets collide. (2 marks)

**Question 19 (7 marks)**

A spherical container of radius  is partly filled with water so that its depth at the deepest point is.

The cross-section of the container is represented by the graph of  .

Show that the ratio of

the water in the partly filled container : water in the full sphere =  .

Does this fraction give the expected answers when  and when ?

**Question 20 (12 marks)**

(a) Use the substitution  to show that .

(3 marks)

(b) Consider the logistic differential equation for a population ,  , where  and  are constants and  is measured in years. Show that the population grows fastest when . (2 marks)

(c) Another model for population is given by the solution of the differential equation



where  is a constant. Show that this population grows fastest when . (2 marks)

(d) Given  and , and that  when , compare the population sizes predicted by the two models when . (5 marks)

**Additional working space**

Question number: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Acknowledgements**

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